Robust Covariance Matrix Estimation and Sparse Bias Estimation for Multipath Mitigation

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BIOGRAPHIES

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Jean-Yves TOURNERET received the ingénieur degree in electrical engineering from the Ecole Nationale Supérieure d'Electronique, d'Electrotechnique, d'Informatique, d'Hydraulique et des Télécommunications (ENSEEIHT) de Toulouse in 1989 and the Ph.D. degree from the National Polytechnic Institute from Toulouse in 1992. He is currently a professor in the university of Toulouse (ENSEEIHT) and a member of the IRIT laboratory (UMR 5505 of the CNRS). His research activities are centered around statistical signal and image processing with a particular interest to Bayesian and Markov chain Monte Carlo (MCMC) methods. He has been involved in the organization of several conferences including the European conference on signal processing EUSIPCO'02 (program chair), the international conference ICASSP'06 (plenaries), the statistical signal processing workshop SSP'12 (international liaisons), the International Workshop on Computational Advances in Multi-Sensor Adaptive Processing CAMSAP 2013 (local arrangements), the statistical signal processing workshop SSP'2014 (special sessions), the workshop on machine learning for signal processing MLSP'2014 (special sessions). He has been the general chair of the CIMI workshop on optimization and statistics in image processing hold in Toulouse in 2013 (with F. Malgouyres and D. Kouamé) and of the International Workshop on Computational Advances in Multi-Sensor Adaptive Processing CAMSAP 2015 (with P. Djuric). He has been a member of different technical committees including the Signal Processing Theory and Methods (SPTM) committee of the IEEE Signal Processing Society (2001-2007, 2010-2015). He has been serving as an associate editor for the IEEE Transactions on Signal Processing (2008-2011, 2015-present) and for the EURASIP journal on Signal Processing (2013-present).

ABSTRACT

Multipath is an important source of error when using global navigation satellite systems (GNSS) in urban environment, leading to biased measurements and thus to false positions. This paper treats the GNSS navigation problem as the resolution of an overdetermined system, which depends on the receiver's position, velocity, clock bias, clock drift, and possible biases affecting GNSS measurements. We investigate a sparse estimation method combined with an extended Kalman filter to solve the navigation problem and estimate the multipath biases. The proposed sparse estimation method assumes that only a part of the satellites are affected by multipath, i.e., that the unknown bias vector is sparse in the sense that several of its components are equal to zero. The natural way of enforcing sparsity is to introduce an ℓ_1 regularization ensuring that the bias vector has zero components. This leads to a least absolute shrinkage

and selection operator (LASSO) problem, which is solved using a reweighted-ℓ_1 algorithm. The weighting matrix of this algorithm is defined as functions of the carrier to noise density ratios and elevations of the different satellites. Moreover, the smooth variations of multipath biases versus time are enforced using a regularization based on total variation. For estimating the noise covariance matrix, we use an iterative reweighted least squares strategy based on the so-called Danish method. The performance of the proposed method is assessed via several simulations conducted on different real datasets.

INTRODUCTION

Multipath (MP) is one of the most difficult error sources that needs to be tackled for GNSS positioning [1]. MP signals are generally due to reflections on various obstacles, and thus strongly depend on the geometric configuration of the scene of interest. More precisely, in the absence of obstacle, the receiver will not suffer from MP. On the contrary, e.g., when the receiver is next to buildings, the received GNSS measurements are very likely to be subjected to MP. The mitigation of MP in GNSS has received a considerable attention in the literature. MP can be mitigated for instance at the antenna level [2] or at the receiver level, more precisely working on the correlator [3, 4] or the discriminator [5]. These techniques require to have access to the receiver's hardware, which is not possible when components on the shelf (COTS) have to be used. Dealing with MP at a measurement or position level is thus an interesting alternative. A first MP mitigation technique consists in exploiting a 3D model of the environment to predict MP signals [6], and to possibly combine this information with measurements acquired by other sensors, such as cameras or IMU (Inertial Measurement Unit). However, these techniques require to have access to an accurate 3D model of the environment at any time instant. A second option is to use the information available at the receiver resulting from pseudoranges, Doppler shifts, satellite ephemeris and C/N_0. Other techniques combine different measurements from the same satellite, e.g., by using the difference between the measurements from two receivers leading to differential GNSS [7, ch. 8] or from two different users (collaborative or cooperative positioning) [8]. An interesting family of MP mitigation methods relies on statistical tests trying to exclude or correct faulty measurements. The receiver autonomous integrity monitoring (RAIM) method belongs to this class of strategies [7, ch. 15]. A more recent strategy based on a-contrario models allows the satellites affected by MP to be excluded from the set of measurements [9]. Note that these techniques require redundant measurements, which can be restrictive in urban environment. Finally, it is interesting to mention other techniques assuming that the presence of MP affects the Gaussiannity of the additive noise, which can be handled using Markov processes [10] or Dirichlet process mixtures [11].

The point of view considered in this work is to model the effect of MP on GNSS measurements as sparse additive biases with temporal smoothing as in [12, 13]) These additive biases are then estimated and subtracted from the GNSS measurements to mitigate MP effects. However, we have observed that this method can be sensitive to the imperfect knowledge of the noise covariance matrix. As a consequence, we propose to use the so-called Danish method [14] to estimate this covariance matrix. The main contribution of this paper is to combine all these ingredients (sparse estimation, temporal smoothing of the biases and estimation of the noise covariance matrix) leading to an improved positioning algorithm.

This paper is organized as follows: Section 1 summarizes some basic principles on satellite navigation, describing how measurements (pseudoranges and pseudorange rates) are related to the state vector (position, velocity, clock bias and clock drift) and to possible MP biases. This section also recalls the Kalman filtering steps that will be used to track the receiver position. Section 2 presents the methods proposed in [12, 13] to estimate MP biases using sparse estimation, formulating the positioning problem as a penalized least squares problem with a weighted ℓ_1 regularization and ℓ_1 smoothing. Section 3 describes the Danish method allowing the noise covariance matrix to be estimated, thanks to an iterative reweighted least squares (IRLS) algorithm. The proposed navigation algorithm is presented in Section 4. Section 5 evaluates the performance of the proposed estimation strategy via experimental results, showing interesting improvements for GNSS navigation.

1 GNSS NAVIGATION

1.1 STATE MODEL

The GNSS navigation problem is formulated using the method described in [7, ch. 7], which is summarized below. The unknown state vector at time k (to be estimated) is defined as $\boldsymbol{X}_k = (x_k, y_k, z_k, b_k, \dot{x}_k, \dot{y}_k, \dot{z}_k, \dot{b}_k)^T$ where $\boldsymbol{r}_k = (x_k, y_k, z_k)^T$ and $\boldsymbol{v}_k = (\dot{x}_k, \dot{y}_k, \dot{z}_k)^T$ are the receiver position and velocity in a given frame, b_k is the receiver clock bias, $\dot{b}_k$ is the receiver clock drift, and the subscript k refers to the kth time instant. A random walk is adopted for the state propagation, leading to

$$\boldsymbol{X}_{k+1} = \boldsymbol{F}_k \boldsymbol{X}_k + \boldsymbol{u}_k \quad \text{with} \quad \boldsymbol{F}_k = \begin{bmatrix} \boldsymbol{I}_4 & (\Delta t_k)\boldsymbol{I}_4 \\ \boldsymbol{0}_4 & \boldsymbol{I}_4 \end{bmatrix} \tag{1}$$

where $\boldsymbol{I}_4$ is the 4×4 identity matrix, $\boldsymbol{0}_4$ is the 4×4 zero matrix, Δt_k is the time between time instants k and $k+1$, and $\boldsymbol{u}_k$ is a Gaussian noise vector of covariance matrix $\boldsymbol{Q}_k \in \mathbb{R}^{8 \times 8}$, i.e.,

$$\boldsymbol{u}_k \sim \mathcal{N}(\boldsymbol{0}_8, \boldsymbol{Q}_k) \tag{2}$$

where $\boldsymbol{0}_8$ is the zero vector of $\mathbb{R}^8$ and $\mathcal{N}(.)$ is the normal distribution (closed-form expressions for $\boldsymbol{Q}_k$ can be found in standard textbooks such as [15, ch. 11], [16, ch. 12]).

1.2 OBSERVATION MODEL

To estimate the unknown state vector $\boldsymbol{X}_k$, we will use two kinds of measurements: the pseudoranges, corresponding to the ranges between the receiver and the satellites, and the pseudorange rates (equal to the Doppler measurements up to a multiplicative constant) corresponding to the relative velocities between the receiver and the satellites. Denoting as s_k the number of satellites that are visible at time instant k, the number of measurements given by the receiver is $2s_k$, namely s_k pseudoranges, denoted as $\rho_{1,k}, ..., \rho_{s_k,k}$, and s_k pseudorange rates, denoted as $\dot{\rho}_{1,k}, ..., \dot{\rho}_{s_k,k}$. These measurements are gathered in the vector $\boldsymbol{z}_k = (z_{k,1}, ..., z_{k,2s_k})^T \in \mathbb{R}^{2s_k}$ whose components are defined as

$$z_{k,i} = \rho_{i,k} \quad \text{and} \quad z_{k,i+s_k} = \dot{\rho}_{i,k} \quad \text{for } i = 1, ..., s_k. \tag{3}$$

These measurements are related to the various components of the state vector since

$$\rho_{i,k} = \|\boldsymbol{r}_k - \boldsymbol{r}_{i,k}\|_2 + b_k + \varepsilon_{i,k} \tag{4}$$

$$\dot{\rho}_{i,k} = (\boldsymbol{v}_k - \boldsymbol{v}_{i,k})^T \frac{\boldsymbol{r}_k - \boldsymbol{r}_{i,k}}{\|\boldsymbol{r}_k - \boldsymbol{r}_{i,k}\|_2} + \dot{b}_k + \dot{\varepsilon}_{i,k} \tag{5}$$

where

- $\boldsymbol{r}_{i,k} = (x_{i,k}, y_{i,k}, z_{i,k})^T$ is the ith satellite position at time k expressed in the same frame as $\boldsymbol{r}_k$,
- $\boldsymbol{v}_{i,k} = (\dot{x}_{i,k}, \dot{y}_{i,k}, \dot{z}_{i,k})^T$ is the ith satellite velocity at time instant k expressed in the same frame as $\boldsymbol{v}_k$,
- $\|\boldsymbol{r}_k - \boldsymbol{r}_{i,k}\|_2 = \sqrt{(x_k - x_{i,k})^2 + (y_k - y_{i,k})^2 + (z_k - z_{i,k})^2}$ is the range between the user and the ith satellite,
- $\varepsilon_{i,k}$ and $\dot{\varepsilon}_{i,k}$ are the error terms associated with the ith propagation channel (accounting for ionospheric delay, tropospheric delay, satellite clock biases, satellite position uncertainties, Sagnac effects, relativistic effects, MP biases and receiver noise).

Note that b_k and $\dot{b}_k$ do not depend on i, and that $\varepsilon_{i,k}$ and $\dot{\varepsilon}_{i,k}$ summarize all the error sources affecting the s_k corresponding measurements. After applying correction models for each error except MP, the measurement equations can be rewritten as

$$\boldsymbol{z}_k = \boldsymbol{h}_k(\boldsymbol{X}_k) + \boldsymbol{m}_k + \boldsymbol{n}_k \tag{6}$$

where $\boldsymbol{m}_k = (m_{1,k}, ..., m_{2s_k,k})^T \in \mathbb{R}^{2s_k}$ is the vector accounting for the potential MP biases, $\boldsymbol{n}_k = (n_{1,k}, ..., n_{2s_k,k})^T \in \mathbb{R}^{2s_k}$ is the receiver noise vector supposed centered and Gaussian with unknown covariance matrix $\boldsymbol{R}_k \in \mathbb{R}^{2s_k \times 2s_k}$ (as discussed in Section 3), and $\boldsymbol{h}_k$ is a nonlinear function which is not explicited here but can be deduced from (4), (5) and error models as in [7, ch. 7]. Provided that the measurement equation is nonlinear, it is natural to use the extended Kalman filter (EKF) [7, ch. 3], [17, ch. 8] to estimate the state vector $\boldsymbol{X}_k$.

1.3 THE EXTENDED KALMAN FILTER FOR NAVIGATION

The EKF consists in applying a Kalman filter to the state equation (1) and the first order approximation around $\hat{\boldsymbol{X}}_{k|k-1}$ (which is the one step prediction of the Kalman filter) of $\boldsymbol{h}_k(\boldsymbol{X}_k)$ in (6),

$$\boldsymbol{z}_k \approx \boldsymbol{h}_k(\hat{\boldsymbol{X}}_{k|k-1}) + \boldsymbol{H}_k(\boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k-1}) + \boldsymbol{m}_k + \boldsymbol{n}_k \tag{7}$$

where $\boldsymbol{H}_k \in \mathbb{R}^{2s_k \times 8}$ is the Jacobian matrix of the function $\boldsymbol{h}_k$ at point $\hat{\boldsymbol{X}}_{k|k-1}$. This measurement equation can be rewritten as

$$\boldsymbol{z}_k - \boldsymbol{h}_k(\hat{\boldsymbol{X}}_{k|k-1}) - \boldsymbol{m}_k = \boldsymbol{H}_k(\boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k-1}) + \boldsymbol{n}_k \tag{8}$$

where the left hand side term is a nonlinear function of the state $\hat{\boldsymbol{X}}_{k|k-1}$. Assuming that $\boldsymbol{m}_k$ is a known bias term and that $\boldsymbol{R}_k$ is a known covariance matrix, the EKF leads to

$$\hat{\boldsymbol{X}}_{k|k-1} = \boldsymbol{F}_{k-1}\hat{\boldsymbol{X}}_{k-1|k-1} \tag{9}$$

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1|k-1}\boldsymbol{F}_{k-1}^T + \boldsymbol{Q}_{k-1} \tag{10}$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T\left(\boldsymbol{H}_k\boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T + \boldsymbol{R}_k\right)^{-1} \tag{11}$$

$$\hat{\boldsymbol{X}}_{k|k} = \hat{\boldsymbol{X}}_{k|k-1} + \boldsymbol{K}_k(\boldsymbol{z}_k - \boldsymbol{h}_k(\hat{\boldsymbol{X}}_{k|k-1}) - \boldsymbol{m}_k) \tag{12}$$

$$\boldsymbol{P}_{k|k} = (I_8 - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_{k|k-1}. \tag{13}$$

The next sections propose a method allowing the unknown bias vector $\boldsymbol{m}_k$ and the unknown covariance matrix $\boldsymbol{R}_k$ to be estimated. These estimated quantities will be used in place of $\boldsymbol{m}_k$ and $\boldsymbol{R}_k$ in (12) and (11).

2 SPARSE ESTIMATION THEORY APPLIED TO GNSS MULTIPATH MITIGATION

This section recalls the principles of the sparse estimation method of [12] using the least absolute shrinkage and selection operator (LASSO) problem and a reweighted-ℓ_1 regularization.

2.1 THE LASSO PROBLEM

Assume that we have a vector of measurements $\tilde{\boldsymbol{y}}_k \in \mathbb{R}^{2s_k}$ defined as $\tilde{\boldsymbol{y}}_k = \tilde{\boldsymbol{H}}_k \boldsymbol{\theta}_k + \tilde{\boldsymbol{n}}_k$, where $\tilde{\boldsymbol{H}}_k \in \mathbb{R}^{2s_k \times 2s_k}$ is a known regression matrix, $\boldsymbol{\theta}_k \in \mathbb{R}^{2s_k}$ is an unknown vector (to be estimated) and $\tilde{\boldsymbol{n}}_k \in \mathbb{R}^{2s_k}$ is an unknown noise term[1]. When $\tilde{\boldsymbol{H}}_k$ is not full rank, the problem is underdetermined, and a classical way of estimating $\boldsymbol{\theta}_k$ from the observed measurement vector $\tilde{\boldsymbol{y}}_k$ is to consider a data fidelity term $\frac{1}{2}\|\tilde{\boldsymbol{y}}_k - \tilde{\boldsymbol{H}}_k \boldsymbol{\theta}_k\|_2^2$ penalized by an appropriate regularization. If one wants to promote the sparsity of $\boldsymbol{\theta}_k$, one can think of defining this regularization as the ℓ_1 norm of $\boldsymbol{\theta}_k$ defined by

$$\|\boldsymbol{\theta}_k\|_1 = \sum_{i=1}^{2s_k} |\theta_{k,i}|. \tag{14}$$

This problem formulation leads to the so-called LASSO estimator defined as [18]

$$\hat{\boldsymbol{\theta}}_k = \underset{\boldsymbol{\theta}_k \in \mathbb{R}^{2s_k}}{\arg\min} \frac{1}{2}\|\tilde{\boldsymbol{y}}_k - \tilde{\boldsymbol{H}}_k \boldsymbol{\theta}_k\|_2^2 + \lambda_k \|\boldsymbol{\theta}_k\|_1 \tag{15}$$

where $\lambda_k \in \mathbb{R}^+$ is a fixed constant referred to as regularization parameter.

2.2 THE REWEIGHTED-ℓ_1 ALGORITHM OF [12]

Candès [19] investigated a so-called reweighted-ℓ_1 method defined as follows

$$\underset{\boldsymbol{\theta}_k \in \mathbb{R}^{2s_k}}{\arg\min} \frac{1}{2}\|\tilde{\boldsymbol{y}}_k - \tilde{\boldsymbol{H}}_k \boldsymbol{\theta}_k\|_2^2 + \lambda_k \|\boldsymbol{W}_k \boldsymbol{\theta}_k\|_1 \tag{16}$$

where $\boldsymbol{W}_k \in \mathbb{R}^{2s_k \times 2s_k}$ is a diagonal weighting matrix. Ideally, the weights contained in $\boldsymbol{W}_k$ should be inversely proportional to the magnitude of the true unknown vector $\boldsymbol{\theta}_k$, i.e., such that

$$w_{k,i} = \begin{cases} \frac{1}{|\theta_{k,i}|}, & \theta_{0,i} \neq 0, \\ \infty, & \theta_{k,i} = 0. \end{cases} \tag{17}$$

However, this weight definition cannot be used in practice since $\boldsymbol{\theta}_k$ is an unknown vector. An iterative solution was studied in [19] to estimate the weights $w_{k,i}$. However, this method did not provide promising results for our application, which motivated us to investigate another reweighting scheme. Looking carefully at (17), we can see that if we have a priori information that $\theta_{k,i}$ has a large (resp. small) value, we should define a low (resp. high) weight $w_{k,i}$. The weighting scheme proposed in the next section takes this observation into account.

2.3 A REWEIGHTED-ℓ_1 METHOD FOR GNSS

In the presence of an additive bias affecting the measurement equation, introducing the notations $\boldsymbol{y}_k = \boldsymbol{z}_k - \boldsymbol{h}_k(\hat{\boldsymbol{X}}_{k|k-1}) \in \mathbb{R}^{2s_k}$ and $\boldsymbol{x}_k = \boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k-1}$, Eq. (7) can be rewritten

$$\boldsymbol{y}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{m}_k + \boldsymbol{n}_k. \tag{18}$$

The proposed MP mitigation method assumes that the bias vector $\boldsymbol{m}_k$ is sparse, i.e., that some of its components are exactly equal to 0. In other words, we assume that among all the satellites, only a few of them suffer from MP. Exploiting this sparsity assumption, we propose to solve the following problem

$$\underset{\boldsymbol{x}_k, \boldsymbol{m}_k}{\arg\min} \frac{1}{2}\|\boldsymbol{y}_k - \boldsymbol{H}_k \boldsymbol{x}_k - \boldsymbol{m}_k\|_2^2 + \lambda_k \|\boldsymbol{W}_k \boldsymbol{m}_k\|_1 \tag{19}$$

in order to estimate the bias vector $\boldsymbol{m}_k$, and feed it to the proposed EKF in (13). Regarding the weighting matrix $\boldsymbol{W}_k$, we propose to consider the strategy of [12], leading to

$$w_1(x) = \begin{cases} 10^{\frac{x-T}{a}} \left(\left(A \times 10^{\frac{F-T}{a}} - 1 \right) \frac{x-T}{F-T} + 1 \right)^{-1}, & x < T \\ 1, & x \geq T \end{cases} \tag{20}$$

where

[1] The meaning of the different vectors $\tilde{\boldsymbol{y}}_k, \boldsymbol{\theta}_k, \tilde{\boldsymbol{n}}_k$ in the GNSS context will be clarified in subsection 2.3.

- x is the value of C/N_0 expressed in dBHz,
- $T = 45$ is a threshold after which the weight is set to 1 (indicating that the measurements are "good"),
- $a = 80$ allows the bending of the curve to be adjusted,
- $F = 20$ defines the value of C/N_0 for which the function w_1 is forced to have the weight defined by parameter A
- $A = 30$ controls the value of the function w_1 for $x = F$ (since $w_1(F) = 1/A$)

and

$$w_2(x) = \begin{cases} \frac{\sin^2(x)}{\sin^2(5^\circ)} & x < 5^\circ \\ 1 & x \geq 5^\circ \end{cases} \tag{21}$$

where x is a given satellite elevation expressed in degrees. The final weight for a given satellite introduced in the reweighted-ℓ_1 approach is defined as the product of the two previous functions, i.e.,

$$w_{i,k}\left[(C/N_0)_{i,k}, e_{i,k}\right] = w_1\left[(C/N_0)_{i,k}\right] w_2(e_{i,k}) \tag{22}$$

where $w_{i,k}$ is the ith diagonal element of the matrix $\boldsymbol{W}_k$, $(C/N_0)_{i,k}$ and $e_{i,k}$ are the C/N_0 and elevation associated with the ith satellite at time instant k. These weights allow us to give more importance to satellites associated with high values of C/N_0 and/or high elevations, since these satellites are less likely to suffer from MP. In order to obtain a formulation similar to (16), it is interesting to note that the minimization of (19) with respect to $\boldsymbol{x}_k$ for a fixed $\boldsymbol{m}_k$ has the following closed-form expression

$$\boldsymbol{x}_k = (\boldsymbol{H}_k^T \boldsymbol{H}_k)^{-1} \boldsymbol{H}_k^T (\boldsymbol{y}_k - \boldsymbol{m}_k) \tag{23}$$

which is the classical least squares solution. After replacing this expression of $\boldsymbol{x}_k$ in (19), we obtain the so-called profile likelihood

$$L(\boldsymbol{m}_k) = \frac{1}{2}\|(I_{2s_k} - \boldsymbol{\Pi}_k)(\boldsymbol{y} - \boldsymbol{m}_k)\|_2^2 + \lambda_k \|\boldsymbol{W}_k \boldsymbol{m}_k\|_1 \tag{24}$$

where $\boldsymbol{\Pi}_k$ is the following projection matrix

$$\boldsymbol{\Pi}_k = \boldsymbol{H}_k (\boldsymbol{H}_k^T \boldsymbol{H}_k)^{-1} \boldsymbol{H}_k^T. \tag{25}$$

Finally, after introducing the following notations

$$\tilde{\boldsymbol{y}}_k = (I_{2s_k} - \boldsymbol{\Pi}_k)\boldsymbol{y}_k \tag{26}$$

$$\tilde{\boldsymbol{H}}_k = (I_{2s_k} - \boldsymbol{\Pi}_k)\boldsymbol{W}_k^{-1} \tag{27}$$

$$\boldsymbol{\theta}_k = \boldsymbol{W}_k \boldsymbol{m}_k \tag{28}$$

the original problem (19) reduces to

$$\underset{\boldsymbol{\theta}_k \in \mathbb{R}^{2s_k}}{\arg\min} \frac{1}{2}\|\tilde{\boldsymbol{y}}_k - \tilde{\boldsymbol{H}}_k \boldsymbol{\theta}_k\|_2^2 + \lambda_k \|\boldsymbol{\theta}_k\|_1. \tag{29}$$

We identify a LASSO problem whose solution can be obtained using classical efficient algorithms [18, 20, 21]. In this paper, we have used the "shooting algorithm" (detailed for instance in [22] and [23]). We have shown in [13] that enforcing smoothness to the MP bias amplitudes provides better local results, as presented in the next subsection.

2.4 SMOOTH SPARSE ESTIMATION FOR GNSS

Based on the fused LASSO described in [24], we proposed in [13] to introduce a penalty associated with the temporal variations of the different biases leading to the following problem

$$\underset{\boldsymbol{\theta}_k \in \mathbb{R}^{2s_k}}{\arg\min} \frac{1}{2}\|\tilde{\boldsymbol{y}}_k - \tilde{\boldsymbol{H}}_k \boldsymbol{\theta}_k\|_2^2 + \lambda_k \|\boldsymbol{\theta}_k\|_1 + \mu_k \|\boldsymbol{\theta}_k - \hat{\boldsymbol{\theta}}_{k-1}\|_1. \tag{30}$$

Note that the temporal smoothing is assigned to the weighted biases and not to the biases themselves. Indeed, this strategy induces more smoothing to channels affected by large weights, which is a desired property.

However, some satellites might not be visible at some time instants k. Thus, the last regularization term has to be only evaluated for satellites that are visible at time instants k and $k-1$. In order to respect this constraint, we introduce the following penalty

$$\|\boldsymbol{\theta}_k - \hat{\boldsymbol{\theta}}_{k-1}\|_{1,\mathcal{S}_k} = \sum_{i \in \mathcal{S}_k} |\theta_{i,k} - \hat{\theta}_{i,k-1}| \tag{31}$$

where $\mathcal{S}_k$ is the set of indices associated with satellites that are jointly visible at time instants k and $k-1$. This leads to the following problem

$$\underset{\boldsymbol{\theta}_k \in \mathbb{R}^{2s_k}}{\arg\min} \frac{1}{2}\|\tilde{\boldsymbol{y}}_k - \tilde{\boldsymbol{H}}_k \boldsymbol{\theta}_k\|_2^2 + \lambda_k \|\boldsymbol{\theta}_k\|_1 + \mu_k \|\boldsymbol{\theta}_k - \hat{\boldsymbol{\theta}}_{k-1}\|_{1,\mathcal{S}_k} \tag{32}$$

3 ROBUST COVARIANCE ESTIMATION: THE DANISH METHOD

The Danish method is an iterative reweighted least squares algorithm based on the deviations between the observations and the measurement model by modifying some a priori variances until some consistency has been achieved. The a priori variances used in this work denoted as $\sigma_{i,0}^2, i = 1, \ldots, 2s_k$ were chosen as in [25]. They allow us to define an initial covariance matrix $\boldsymbol{R}_0 = \text{diag}(\sigma_{i,0}^2)_{i=1,\ldots,2s_k}$, which is used to define the covariance matrix of the residuals at time instant k

$$\boldsymbol{C}_k = \boldsymbol{R}_0 - \boldsymbol{H}_k(\boldsymbol{H}_k^T \boldsymbol{R}_0^{-1} \boldsymbol{H}_k)^{-1} \boldsymbol{H}_k^T. \tag{33}$$

Each value of the variance is then updated iteratively as a function of the corresponding residuals, where l refers to the current iteration and $i = 1, \ldots, 2s_k$

$$\sigma_{i,l+1}^2 = \sigma_{i,0}^2 \times \begin{cases} \exp\left(\frac{w_{i,l}}{T}\right) & \text{if } w_{i,l} > T \\ 1 & \text{otherwise} \end{cases} \tag{34}$$

with

$$w_{i,l} = \left| \frac{\hat{\boldsymbol{v}}_{i,l}}{\sqrt{C_{k,i}}} \right| \tag{35}$$

$$\hat{\boldsymbol{v}}_{i,l} = \left[I - \boldsymbol{H}_k(\boldsymbol{H}_k^T \boldsymbol{W}_l \boldsymbol{H}_k)^{-1} \boldsymbol{H}_k^T \boldsymbol{W}_l \right] (\boldsymbol{y}_k - \hat{\boldsymbol{m}}_k) \tag{36}$$

$$\boldsymbol{W}_l = \text{diag}\left(\frac{1}{\sigma_{i,l}^2} \right) \tag{37}$$

$$T = F^{-1}\left(1 - \frac{\alpha_0}{2}\right) \tag{38}$$

where $C_{k,i}$ is the ith diagonal element of $\boldsymbol{C}_k$, the weights $w_{i,l}$ are the normalized residual updates after each iteration l, F is the inverse distribution function of a $\mathcal{N}(0,1)$ normal distribution, and α_0 is the desired probability of false alarm (chosen as $\alpha_0 = 0.02$ in this paper). Note that the iterations (34) to (37) are made until some stopping criterion is satisfied (e.g., when the variances have converged or when a maximum number of iterations has been reached). The noise covariance matrix is finally estimated as

$$\hat{\boldsymbol{R}}_k = \text{diag}(\sigma_{i,l+1}^2)_{i=1,\ldots,2s_k}. \tag{39}$$

4 PROPOSED NAVIGATION ALGORITHM

The proposed navigation method estimates the MP bias vector thanks to the proposed weighted sparse regularizations (with and without smoothing, as in Section 2.3 and 2.4), uses the Danish method to estimate the noise covariance matrix as in Section 3, and finally replaces these estimates into the EKF presented in Section 1.3. The final proposed strategy can be summarized as

1. estimate the unknown parameter vector $\hat{\boldsymbol{\theta}}_k$ as the solution of

$$\underset{\boldsymbol{\theta}_k \in \mathbb{R}^{2s_k}}{\arg\min} \frac{1}{2} \|\tilde{\boldsymbol{y}}_k - \tilde{\boldsymbol{H}}_k \boldsymbol{\theta}_k\|_2^2 + \lambda_k \|\boldsymbol{\theta}_k\|_1 \tag{40}$$

   or[2]

$$\underset{\boldsymbol{\theta}_k \in \mathbb{R}^{2s_k}}{\arg\min} \frac{1}{2} \|\tilde{\boldsymbol{y}}_k - \tilde{\boldsymbol{H}}_k \boldsymbol{\theta}_k\|_2^2 + \lambda_k \|\boldsymbol{\theta}_k\|_1 + \mu_k \|\boldsymbol{\theta}_k - \hat{\boldsymbol{\theta}}_{k-1}\|_{1,\mathcal{S}_k} \tag{41}$$

2. estimate the bias vector as $\hat{\boldsymbol{m}}_k = \boldsymbol{W}_k^{-1} \hat{\boldsymbol{\theta}}_k$,
3. estimate the noise covariance matrix as $\hat{\boldsymbol{R}}_k$ with the Danish method,
4. consider the EKF proposed in Section 1.3 with equations (9), (10), (11), (12) and (13).

5 EXPERIMENTAL RESULTS

To appreciate the efficiency of the proposed method and evaluate the interest of temporal smoothing, the navigation algorithm was applied to real measurements provided by a Ublox AEK-4T receiver with and without temporal smoothing. The obtained results are compared with the standard EKF (no bias estimation, no covariance estimation), the Danish method alone (no bias estimation, covariance estimation thanks to the Danish method), the solution investigated in [12] (bias estimation thanks to sparse regularization without

[2] see [26] for an example of algorithm to solve this problem

temporal smoothing, no covariance estimation), referred to as Weighted LASSO, the solution investigated in [13] (bias estimation thanks to sparse regularization with temporal smoothing, no covariance estimation), referred to as Fused LASSO, and the solution provided by the receiver (which is a black box), referred to as Ublox. A reference solution was obtained during the measurement campaign using a very accurate receiver, i.e., a Novatel SPAN composed of a GPS receiver Propak-V3 and an inertial measurement unit (IMAR).

We decided to divide the whole campaign into two parts: the first part corresponds to few MP (sparsity assumption is then appropriate), referred to as "light urban" and the second part is a more urban trajectory (where the sparsity assumption might fail), referred to as "deep urban". Note that the regularization parameters λ_k and μ_k were fixed by cross-validation leading to $(\lambda_k, \mu_k) = (1, 1)$. The whole trajectory is depicted in Fig. 1. As one can see, this trajectory covers various scenarios from open sky (bottom) to deep urban (top). The planar and altitude errors (versus time) are displayed in Fig. 2, and the corresponding cumulative distribution functions (CDFs) are shown in Fig. 3. Finally, some error percentiles have been summarized in Tab. 1. The CDF plots show that the solutions combining the weighted LASSO and the Danish method, and the Fused LASSO and the Danish method yield the best results for both types of error, except for the maximum altitude error (whose minimum is reached by the Ublox). We can also notice that the standard EKF does not provide good localization performance.

Figure 1: Studied trajectory.

Planar	50%	67%	75%	95%	99%	Max
Standard EKF	6.25	11.72	15.10	29.75	41.19	68.487
Weighted LASSO	3.46	4.74	5.79	16.45	28.27	52.059
Fused LASSO	3.439	4.69	5.63	16.06	26.66	39.21
Danish	3.59	4.68	5.539	13.75	23.48	42.29
Weighted LASSO + Danish	3.31	4.32	4.91	12.71	21.86	42.58
Fused LASSO +Danish	3.34	4.33	9 4.97	12.27	21.20	44.46
Ublox	3.78	5.17	6.36	13.67	22.68	45.58
Horizontal	50%	67%	75%	95%	99%	Max
Standard EKF	5.96	10	15.54	39.34	73.47	88.61
Weighted LASSO	5.13	6.06	6.66	14.04	26.93	49.11
Fused LASSO	5.03	5.89	6.45	13.17	26.36	35.32
Danish	5.46	6.19	6.71	12.51	18.05	33.27
Weighted LASSO + Danish	5.71	6.58	7.03	11.57	18.47	32.27
Fused LASSO +Danish	5.57	6.41	6.85	12.62	17.97	32.64
Ublox	10.05	12.65	14.08	16.55	17.24	22.73

Table 1: Chosen percentile of planar errors (top) and altitude errors (bottom) for the different methods (in meters).

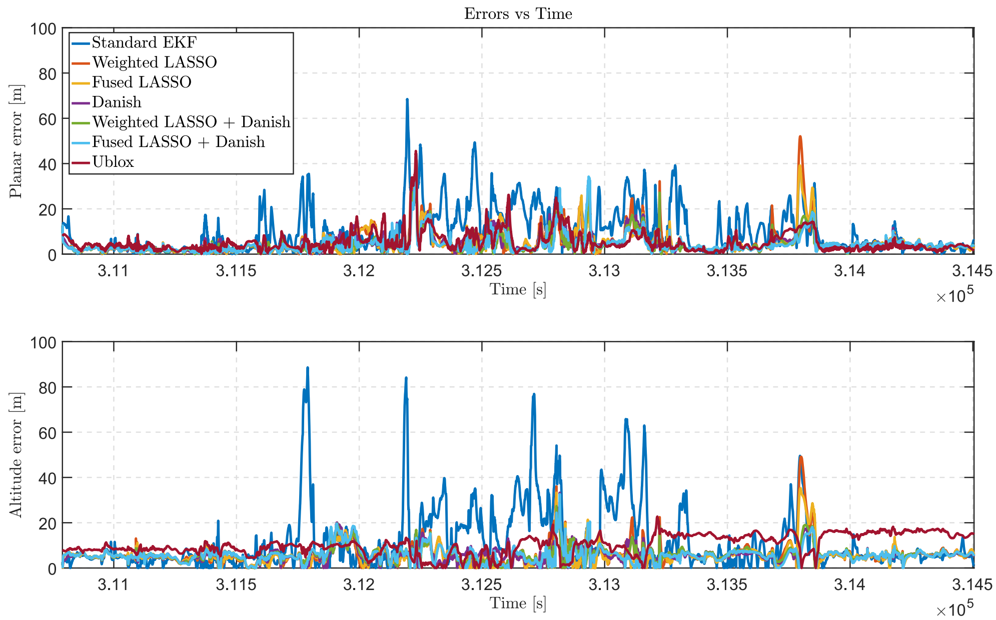

Figure 2: Estimated planar (top) and altitude (bottom) errors versus time for the different methods.

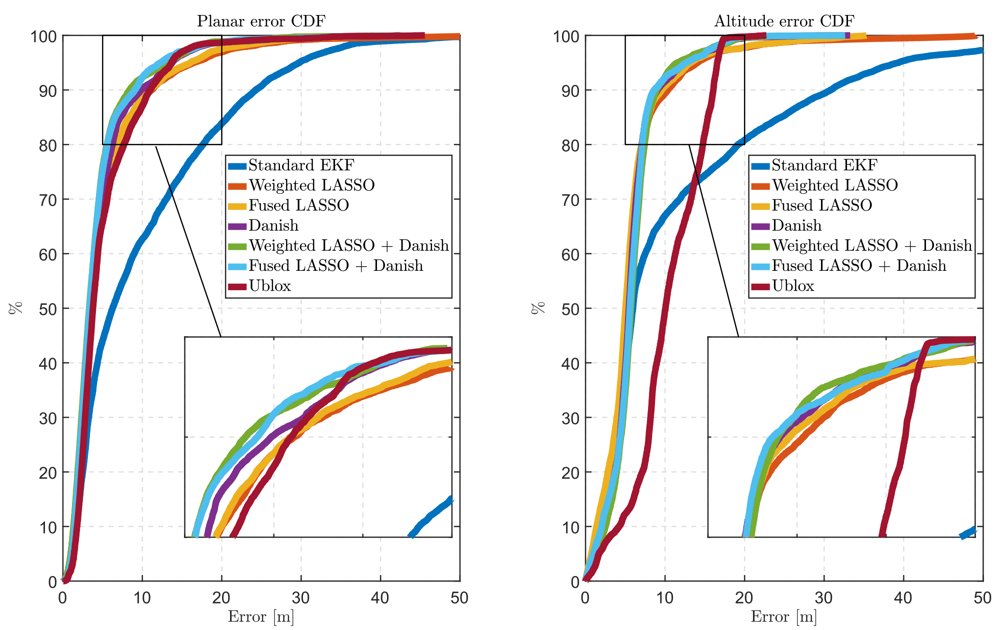

Figure 3: Estimated planar (left) and altitude (right) error CDFs for the different methods.

To better characterize all these solutions, we have divided the whole campaign into two parts and have evaluated the navigation performance for these two parts. The trajectory corresponding to a light urban scenario is depicted in Fig. 4. The corresponding planar and altitude errors (versus time) are shown in Fig. 5, whereas the corresponding CDFs are displayed in Fig. 6. All methods outperform the standard EKF, as the error is not centered white Gaussian. Note that all the proposed methods seem to be equivalent in terms of CDFs. However, the Fused LASSO + Danish provides the best results with the lower maximum error.

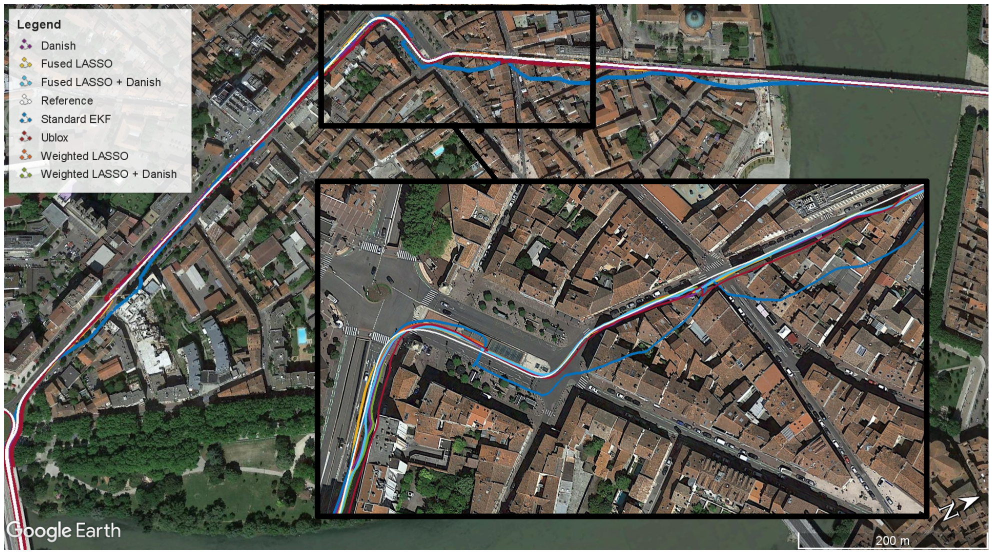

Figure 4: Part of the trajectory with few MP and zoom.

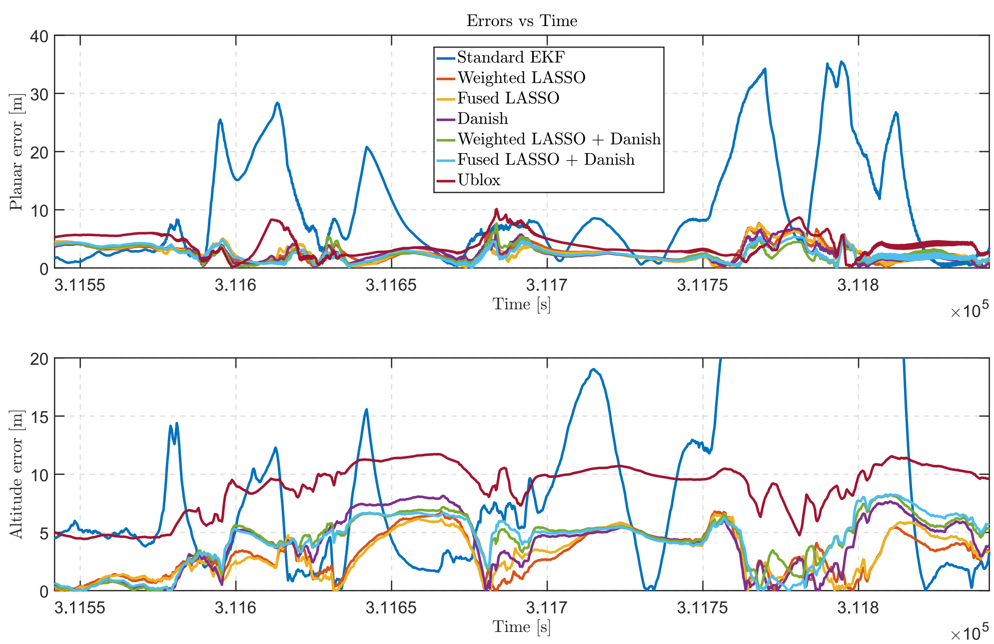

Figure 5: Estimated planar (top) and altitude (bottom) errors versus time for the different methods for the light urban scenario.

The trajectory corresponding to a deep urban scenario is depicted in Fig. 7. The planar and altitude errors and the corresponding CDFs are shown in Figs. 8 and 9. The solution given by the Ublox receiver provides the smallest planar error. Indeed, in such environments, the sparsity assumption fails and the algorithms are not able to estimate all the biases. However, all the proposed solutions outperform the standard EKF. Moreover, the proposed Weighted LASSO + Danish performs better than the Danish method, hence the benefits of estimating the biases due to MP. It is interesting to note that the Fused LASSO + Danish does not provide good results. This could be due to the fact that discontinuities of the MP biases in such environments are not contradictory and there is no need to introduce a temporal smoothing for this part of the trajectory.

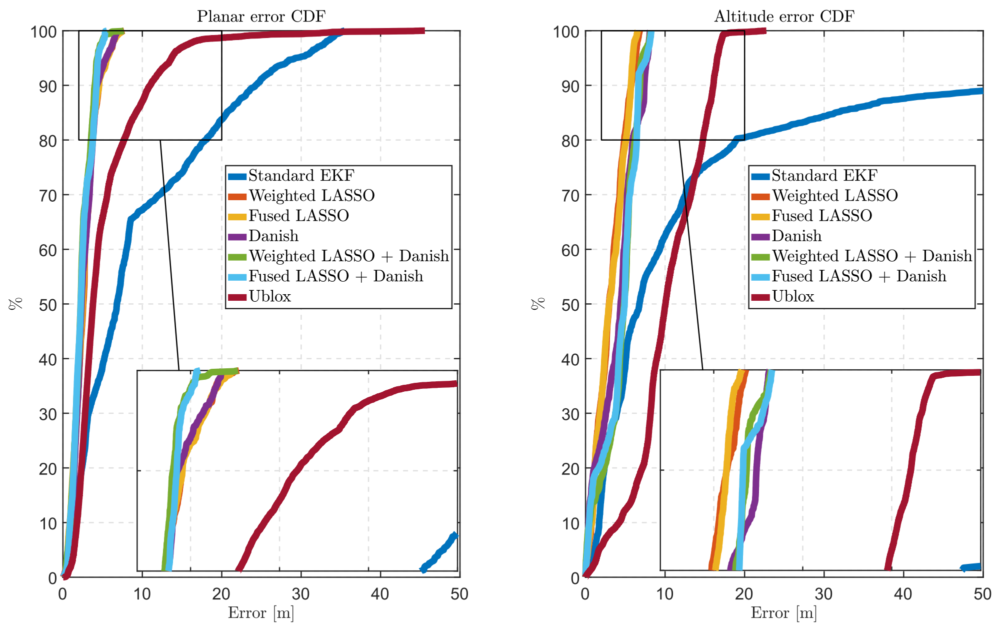

Figure 6: Estimated planar (left) and altitude (right) error CDFs for the different methods for the light urban scenario.

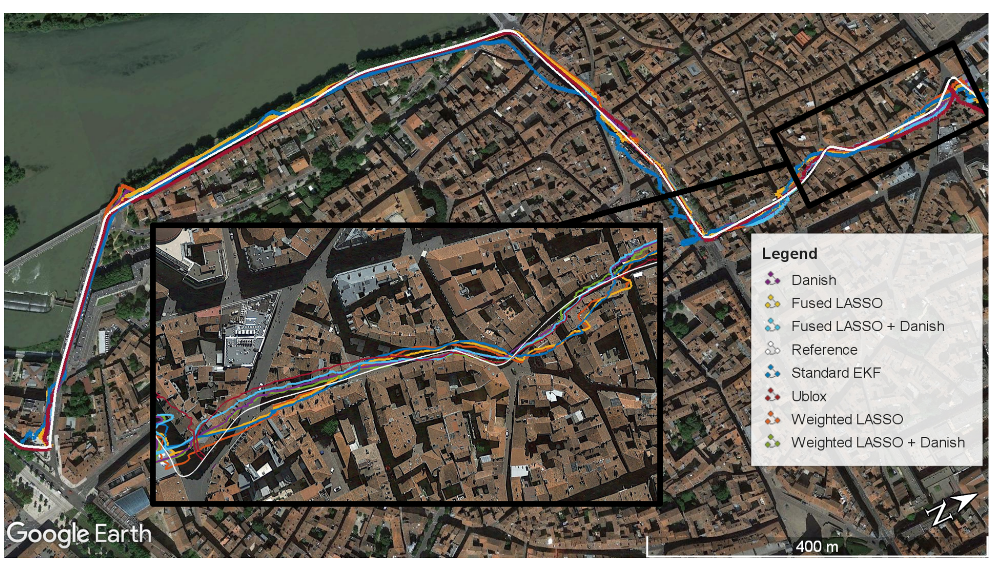

Figure 7: Studied trajectory with supposed many MP and zoom.

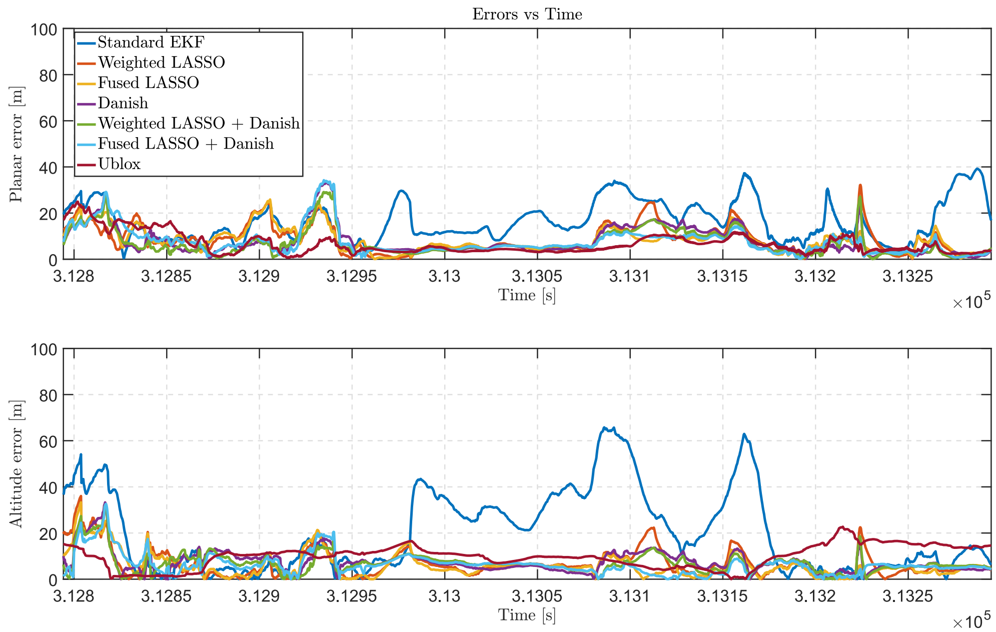

Figure 8: Estimated planar (top) and altitude (bottom) errors CDFs versus time for the different methods for the deep urban scenario.

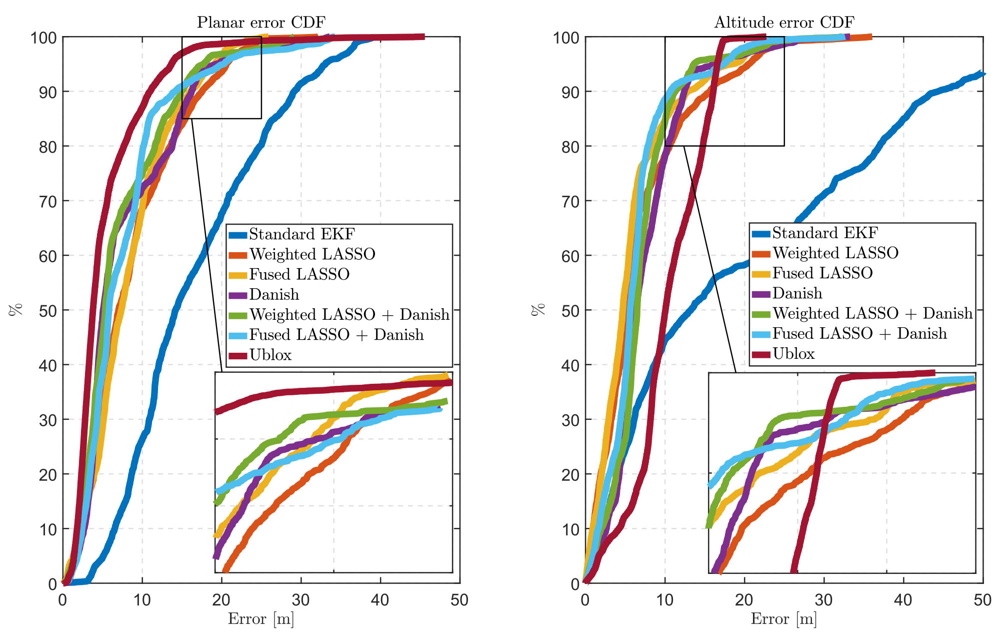

Figure 9: Estimated planar (left) and altitude (right) error CDFs for the different methods for the deep urban scenario.

CONCLUSION

This paper investigated a modification of the reweighted-ℓ_1 method investigated in [12] and its smoothed version introduced in [13] to mitigate MP effects for GNSS navigation. The proposed modified algorithm exploits the joint smoothness and sparsity properties of MP affecting the different satellite channels with an estimation of the measurement noise covariance matrix based on the Danish method. Experiments conducted on real data clearly outlined the benefits of estimating jointly the measurement noise covariance matrix, multipath biases and the position-clock bias state vector. Although the sparsity assumption fails in very harsh environments such as urban canyons, the proposed method performs better than the standard extended Kalman filter. As future work, we think that it might be interesting to develop algorithms switching between methods using smoothing or not, depending on the type of urban scenario (resp. light or deep).

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