Multipath Mitigation for GNSS Positioning in Urban Environment Using Sparse Estimation - Supplementary Material

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Abstract

This document contains supplementary materials associated with the journal paper [1].

1 Introduction

The aim of the paper [1] was to introduce a sparse estimation method for the multipath (MP) biases in Global Navigation Satellite Systems (GNSS) measurements (pseudoranges and pseudorange rates). To do so, we supposed the receiver has N satellites in view, and thus has access to $2N$ measurements (N pseudoranges and N pseudorange rates) which can be expressed as the linearized problem

$$\boldsymbol{y}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{m}_k + \boldsymbol{n}_k \tag{1}$$

where

- $\boldsymbol{y}_k \in \mathbb{R}^{2N}$ contains the difference between the actual and predicted pseudoranges and pseudorange rates at time k,
- $\boldsymbol{H}_k$ is the joint observation matrix for pseudoranges and pseudorange rates at time k described in the next section,
- $\boldsymbol{x}_k = \boldsymbol{s}_k - \hat{\boldsymbol{s}}_k \in \mathbb{R}^8$ is the difference between the state vector (receiver position, velocity, clock bias, and clock drift) estimated at the previous position and the current state vector at time k,
- $\boldsymbol{m}_k \in \mathbb{R}^{2N}$ is a bias term due to the possible presence of MP at time k,
- $\boldsymbol{n}_k \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{R}_k) \in \mathbb{R}^{2N}$ is a zero-mean Gaussian noise vector at time k with covariance matrix $\boldsymbol{R}_k$, described in the next section.

2 Matrices used for the state and measurement equations of the Kalman filter

This section provides the expressions of the matrices used in the state and measurement equations of the Kalman filter considered for target tracking. We start with the Jacobian matrix $\boldsymbol{H}_k$. Let $\boldsymbol{x}_{k-1}$ be the position where the problem is linearized and $\boldsymbol{x}_k^i$ the position of the i-th satellite at time instant k. It is shown in the GNSS literature [2, 3] that if we use the notation

$$\begin{aligned} \boldsymbol{a}_k^i &= -\frac{(\boldsymbol{x}_k^i - \boldsymbol{x}_{k-1})}{\|\boldsymbol{x}_k^i - \boldsymbol{x}_{k-1}\|_2} && (2) \\ &= \begin{bmatrix} a_1^i, & a_2^i, & a_3^i \end{bmatrix}^T && (3) \end{aligned}$$

the Jacobian matrix $\boldsymbol{H}_k$ for the linearized problem (1) is

$$\boldsymbol{H}_k = \begin{bmatrix} a_1^1 & 0 & a_2^1 & 0 & a_3^1 & 0 & 1 & 0 \\ a_1^2 & 0 & a_2^2 & 0 & a_3^2 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_1^N & 0 & a_2^N & 0 & a_3^N & 0 & 1 & 0 \\ 0 & a_1^1 & 0 & a_2^1 & 0 & a_3^1 & 0 & 1 \\ 0 & a_1^2 & 0 & a_2^2 & 0 & a_3^2 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & a_1^N & 0 & a_2^N & 0 & a_3^N & 0 & 1 \end{bmatrix}. \tag{4}$$

Regarding the process covariance matrix, if we consider the closed form expression given in [4, 5], which requires the parameters

- σ_a^2, the variance of the user acceleration ($\sigma_a = 2$ m.s^{-2}),
- σ_b^2, the variance of the user clock bias ($\sigma_b^2 = 0.5 \times 2 \times 10^{-19}$ s^2),
- σ_d^2, the variance of the user clock drift ($\sigma_d^2 = 2\pi^2 \times 2 \times 10^{-20}$),
- Δt, the time gap between where the linearization is made

and the notations

- $\boldsymbol{Q}_{x,k} = \begin{bmatrix} \sigma_a^2 \frac{\Delta t^3}{3} & \sigma_a^2 \frac{\Delta t^2}{2} \\ \sigma_a^2 \frac{\Delta t^2}{2} & \sigma_a^2 \Delta t \end{bmatrix}$,
- $\boldsymbol{Q}_{b,k} = \begin{bmatrix} c^2 \left(\sigma_b^2 \Delta t + \sigma_d^2 \frac{\Delta t^3}{3}\right) & c^2 \left(\sigma_d^2 \frac{\Delta t^2}{2}\right) \\ c^2 \left(\sigma_d^2 \frac{\Delta t^2}{2}\right) & c^2 \left(\sigma_d^2 \Delta t\right); \end{bmatrix}$

we have

$$\boldsymbol{Q}_k = \begin{bmatrix} \boldsymbol{Q}_{x,k} & \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} \\ \mathbf{0}_{2\times2} & \boldsymbol{Q}_{x,k} & \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} \\ \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} & \boldsymbol{Q}_{x,k} & \mathbf{0}_{2\times2} \\ \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} & \boldsymbol{Q}_{b,k} \end{bmatrix}. \tag{5}$$

Finally, the measurement noise covariance matrix $\boldsymbol{R}_k$ was defined as

$$\boldsymbol{R}_k = \begin{bmatrix} \sigma_{\mathrm{UERE}}^2 I_N & \mathbf{0} \\ \mathbf{0} & \dot{\sigma}_{\mathrm{UERE}}^2 I_N \end{bmatrix} \tag{6}$$

with $\sigma_{\mathrm{UERE}} = 5$ m and $\dot{\sigma}_{\mathrm{UERE}} = \lambda_{L_1} \times (2$ Hz$) \approx 0.38$ m.s^{-1}, with λ_{L_1} the wavelength of the GPS signals (UERE stands for User Equivalent Range Error).

3 An algorithm for solving the LASSO problem

The LASSO problem

$$\arg\min_{\boldsymbol{\theta}\in\mathbb{R}^q} \frac{1}{2}\|\tilde{\boldsymbol{y}} - \tilde{\boldsymbol{H}}\boldsymbol{\theta}\|_2^2 + \lambda\|\boldsymbol{\theta}\|_1 \tag{7}$$

can be solved using several algorithms [6, 7, 8]. For this method, we used the so-called shooting algorithm. First, assume that $\boldsymbol{\theta} = \theta$ is of dimension $q = 1$. Hence $\tilde{\boldsymbol{H}} = \tilde{\boldsymbol{h}} \in \mathbb{R}^{2N\times 1}$ and (7) becomes

$$\arg\min_{\theta\in\mathbb{R}} \frac{1}{2}\sum_{i=1}^{N}(\tilde{y}_i - \tilde{h}_i\theta)^2 + \lambda|\theta| \tag{8}$$

If we assume $\theta \geq 0$, the absolute value in (8) disappears, and the resulting expression can be differentiated with respect to θ leading to

$$\sum_{i=1}^{N} -\tilde{h}_i(\tilde{y}_i - \tilde{h}_i\theta) + \lambda = 0 \tag{9}$$

or equivalently

$$\theta = \frac{\sum_{i=1}^{N}\tilde{h}_i\tilde{y}_i - \lambda}{\sum_{i=1}^{N}\tilde{h}_i^2} \tag{10}$$

with the condition $\theta \geq 0$, i.e., $\sum_{i=1}^{N}\tilde{h}_i\tilde{y}_i > \lambda$. The same operation can be conducted with the hypothesis $\theta \leq 0$ to find the general expression of θ

$$\theta = \frac{1}{\tilde{\boldsymbol{h}}^T\tilde{\boldsymbol{h}}}\mathcal{S}_\lambda(\tilde{\boldsymbol{h}}^T\tilde{\boldsymbol{y}}) \tag{11}$$

where $\mathcal{S}_\lambda$ is the soft thresholding operation

$$\mathcal{S}_\lambda(x) = \begin{cases} x-\lambda & \text{if } x > \lambda \\ x+\lambda & \text{if } x < -\lambda \\ 0 & \text{else} \end{cases} \tag{12}$$

Now we can consider the case where $\tilde{\boldsymbol{H}} \in \mathbb{R}^{2N\times q}$. An efficient algorithmic is obtained by adjusting the coordinates of $\boldsymbol{\theta}$ iteratively coordinatewise. Assume we want to estimate the jth coordinate of $\boldsymbol{\theta}$. The problem (7) can be rewritten

$$\arg\min_{\boldsymbol{\theta}\in\mathbb{R}^q} \frac{1}{2}\sum_{i=1}^{N}\left(\tilde{y}_i - \sum_{k\neq j}\tilde{h}_{ik}\theta_k - \tilde{h}_{ij}\theta_j\right)^2 + \lambda\sum_{k\neq j}|\theta_k| + \lambda|\theta_j|. \tag{13}$$

Solving for θ_j and using (11) yields

$$\theta_j = \frac{1}{\tilde{\boldsymbol{h}}_j^T\tilde{\boldsymbol{h}}_j}\mathcal{S}_\lambda(\tilde{\boldsymbol{h}}_j^T(\tilde{\boldsymbol{y}} - \tilde{\boldsymbol{H}}^{(-j)}\boldsymbol{\theta}^{(-j)})) \tag{14}$$

where

- $\tilde{\boldsymbol{h}}_j$ is the jth column of $\tilde{\boldsymbol{H}}$,
- $\tilde{\boldsymbol{H}}^{(-j)}$ is the matrix $\tilde{\boldsymbol{H}}$ with jth column replaced by $\mathbf{0}$,
- $\boldsymbol{\theta}^{(-j)}$ is the vector $\boldsymbol{\theta}$ whose jth element has been replaced by 0.

This step is conducted for each of the N elements of $\boldsymbol{\theta}$ successively until convergence (ensured by the convexity of the objective function $\boldsymbol{\theta} \mapsto \frac{1}{2}\|\tilde{\boldsymbol{y}} - \tilde{\boldsymbol{H}}\boldsymbol{\theta}\|_2^2 + \lambda\|\boldsymbol{\theta}\|_1$).

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